3.11 SOLUTIONS

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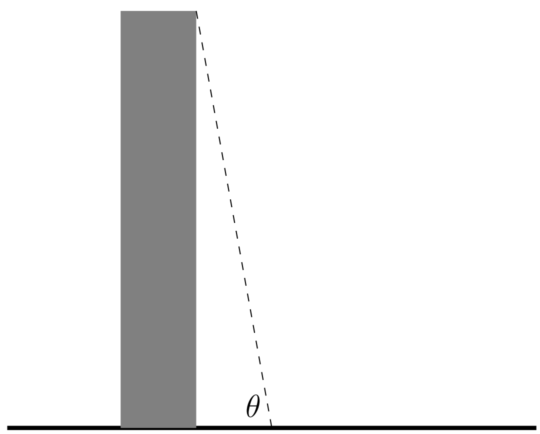
52. The circumference of the tree C of radius r is given by $C = 2\pi r$. We compute the differential $dC = 2\pi dr$. If the circumference increased by 2 inches, then we solve $2 = 2\pi dr$ to find $dr = \frac{1}{\pi}$. In other words, the radius changed by approximately $\frac{1}{\pi}$ inches. It follows that the diameter changes by approximately $\frac{2}{\pi}$ inches.

The tree's cross-sectional area A is given by $A = \pi r^2$. We compute the differential $dA = 2\pi r dr = C dr$ to get an estimate of the area change. We plug in $C = 10$ and $dr = \frac{1}{\pi}$ to see that the area changed by approximately $\frac{10}{\pi}$ square inches.



54.

A surveyor standing 30 ft away from the base of the building, measures the angle of elevation of the top of the building to be 75°. We want to know how accurately we need to measure the angle for the percentage error in estimating the height of the building to be less than 4%.

In all the computations below, θ will be the angle of elevation, measured in radians. The if the answer is desired in radians, we can convert it at the end. We want to use radians so that the derivatives of trig functions are the ones we have memorized. Let h denote the height of the building, measured in feet. Then $\tan\theta = \frac{h}{30}$ so that

$$h = 30\tan\theta \quad \text{and} \quad dh = 30\sec^2\theta d\theta.$$

It follows that the percentage error is approximately

$$\frac{dh}{h} = \frac{\cancel{30}\sec^2\theta d\theta}{\cancel{30}\tan\theta} = \frac{\frac{1}{\cos^2\theta}d\theta}{\frac{\sin\theta}{\cos\theta}} = \frac{d\theta}{\sin\theta\cos\theta}.$$

We want this percentage error less than 4%, so $\frac{d\theta}{\sin\theta\cos\theta} < 0.04$. It follows that (using double-angle formula) we want

$$d\theta < 0.04\sin\theta\cos\theta = 0.02\sin(2\theta).$$

Since $150^\circ = 150(\frac{\pi}{180}) = \frac{5\pi}{6}$ radians, we want $d\theta < 0.02 \sin\left(\frac{5\pi}{6}\right) = 0.02(\frac{1}{2}) = 0.01$. In other words, we need to measure the angle to within 0.01 radians, or

$$0.01\left(\frac{180}{\pi}\right) \approx 0.57295779513082320876798154814 1^\circ.$$

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