2.6 (2, 4, 6, 10, 12, 14, 16, 18, 20, 22, 24, 28, 38, 40, 42, 44, $\boxed{\ }$
58, 64, 66, 68, 70, 80, 90)

② a) 2

b) $\cancel{\ }$ -3

c) 1

d) undefined

e) ∞

f) ∞

g) ∞

h) ∞

i) $-\infty$

j) undefined

k) 0

l) -1

④ $$\lim_{x \to \infty} \pi - \frac{2}{x^2} = \pi - \lim_{x \to \infty} \frac{2}{x^2}$$

$$= \pi$$

⑥ $$\lim_{x \to \infty} \frac{1}{8 - (5/x^2)} = \frac{1}{\lim_{x \to \infty} (8 - 5/x^2)} = \frac{1}{8 - 0} = \frac{1}{8}$$

⑩ $$\lim_{\theta \to -\infty} \frac{\cos \theta}{3\theta} = 0$$

Squeeze between $\frac{-1}{3\theta}$ and $\frac{1}{3\theta}$ to see it.

(12) $\lim_{r \to \infty} \dfrac{r + \sin r}{2r+7-5\sin r}$

Note that $-1 \le \sin r \le 1$

It follows that $\dfrac{r+\sin r}{2r+7-5\sin r}$ is always between

$$\frac{r-1}{2r+7+5} \le \frac{r+\sin r}{2r+7-5\sin r} \le \frac{r+1}{2r+7-5}$$

~~by Squeez~~ $\lim_{r\to\infty} \dfrac{r-1}{2r+12} = \dfrac{1}{2} = \lim_{r\to\infty} \dfrac{r+1}{2r+2}$

By Squeeze Theorem $\lim_{r\to\infty} \dfrac{r+\sin r}{2r+7-5\sin r} = \dfrac{1}{2}$.

(14)
a) $\lim_{x\to\infty} \dfrac{2x^3+7}{x^3-x^2+x+7} = \lim_{x\to\infty} \dfrac{2\cancel{x^3}}{\cancel{x^3}} = 2$

b) similarly $\lim_{x\to-\infty} \dfrac{2x^3+7}{x^3-x^2+x+7} = 2$.

(16) a) $\lim_{x\to\infty} \dfrac{3x+7}{x^2-2} = \lim_{x\to\infty} \dfrac{3\cancel{x}}{x^{\cancel{2}}} = \lim_{x\to\infty} \dfrac{3}{x} = 0$.

b) $\lim_{x\to-\infty} \dfrac{3x+7}{x^2-2} = \lim_{x\to-\infty} \dfrac{3\cancel{x}}{x^{\cancel{2}}} = \lim_{x\to-\infty} \dfrac{3}{x} = 0$.

(18)
a) $\lim_{x\to\infty} \dfrac{1}{x^3-4x+1} = \lim_{x\to\infty} \dfrac{1}{x^3} = 0$.

b) $\lim_{x\to\infty} \dfrac{1}{x^3-4x+1} = \lim_{x\to-\infty} \dfrac{1}{x^3} = 0$.

(20) a) $\lim_{x\to\infty} \dfrac{9x^4+x}{2x^4+5x^2-x+6} = \lim_{x\to\infty} \dfrac{9\cancel{x^4}}{2\cancel{x^4}} = \dfrac{9}{2}$

b) $\lim_{x\to\infty} \dfrac{9x^4+x}{2x^4+5x^2-x+6} = \lim_{x\to-\infty} \dfrac{9\cancel{x^4}}{2\cancel{x^4}} = \dfrac{9}{2}$

(22) a) $\lim_{x \to \infty} \frac{-x^4}{x^4-7x^3+7x^2+9} = \lim_{x \to \infty} \frac{\cancel{-x^4}}{\cancel{x^4}} = -1$

b) $\lim_{x \to -\infty} \frac{-x^4}{x^4-7x^3+7x^2+9} = \lim_{x \to \infty} \frac{\cancel{-x^4}}{\cancel{x^4}} = -1$

(24) $\lim_{x \to -\infty} \left(\frac{x^2+x-1}{8x^2-3} \right)^{1/3} = \left(\lim_{x \to -\infty} \frac{x^2+x-1}{8x^2-3} \right)^{1/3}$

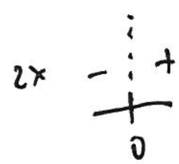
$= \left(\lim_{x \to -\infty} \frac{\cancel{x^2}}{8\cancel{x^2}} \right)^{1/3}$

$= \left(\frac{1}{8} \right)^{1/3}$

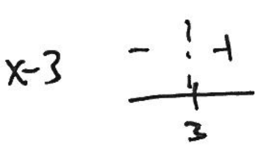
$= \frac{1}{2}$

(28) $\lim_{x \to \infty} \frac{2+\sqrt{x}}{2-\sqrt{x}} = \lim_{x \to \infty} \frac{\cancel{\sqrt{x}}}{-\cancel{\sqrt{x}}} = -1$

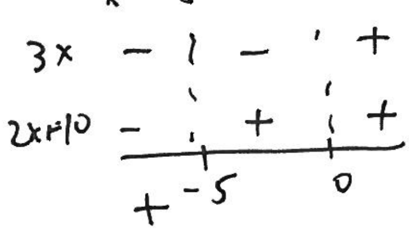
(38) $\lim_{x \to 0^-} \frac{5}{2x} = -\infty$



(40) $\lim_{x \to 3^+} \frac{1}{x-3} = \infty$



(42) $\lim_{x \to -5^-} \frac{3x}{2x+10} = \infty$



(49) $$\lim_{x \to 0} \frac{-1}{x^2(x+1)} = -\infty$$

		-1		0	
-1	$-$		$-$		$-$
$x+1$	$-$		$+$		$+$
x^2	$+$		$+$		$+$
	$+$		$-$		$-$

(58) $$f(x) = \frac{x^2 - 3x + 2}{x^3 - 4x} = \frac{(x-2)(x-1)}{x(x+2)(x-2)}$$

a) $$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{x-1}{x(x+2)} = \frac{2-1}{2(2+2)} = \frac{1}{8}$$

b) $$\lim_{x \to -2^+} f(x)$$

since denominator vanishes but numerator does not vanish, the limit is $\pm\infty$. to see which one, make a sign chart.

		-2		0		1		2	
$x-2$	$-$		$-$		$-$		$-$		$+$
$x-1$	$-$		$-$		$-$		$+$		$+$
x	$-$		$-$		$+$		$+$		$+$
$x+2$	$-$		$+$		$+$		$+$		$+$
$x-2$	$-$		$-$		$-$		$-$		$+$
	$-$		$+$		$-$		$+$		$+$

$$\lim_{x \to -2^+} f(x) = \infty$$

c) $$\lim_{x \to 0^-} f(x) = \infty$$ (see b)

d) $$\lim_{x \to 1^+} f(x) = 0.$$ f is rational function and 1 is in its domain.

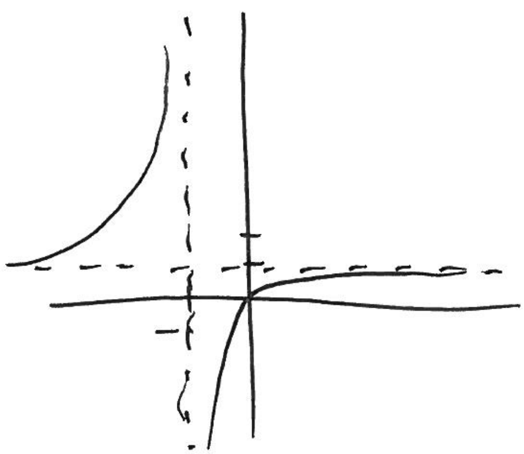
e) $\lim_{x \to 0} f(x)$ is undefined since $\lim_{x \to 0^+} f(x) = -\infty$, $\lim_{x \to 0^-} f(x) = \infty$

(68) $y = \dfrac{2x}{x+1}$ dominant term 2.

$$\begin{array}{r} 2 - \frac{2}{x+1} \\ x+1 \,\overline{)\, 2x + 0} \\ -(2x+2) \\ \hline -2 \end{array}$$

$\lim\limits_{x\to\infty} \dfrac{2x}{x+1} = 2$ horizontal asymptote $y=2$

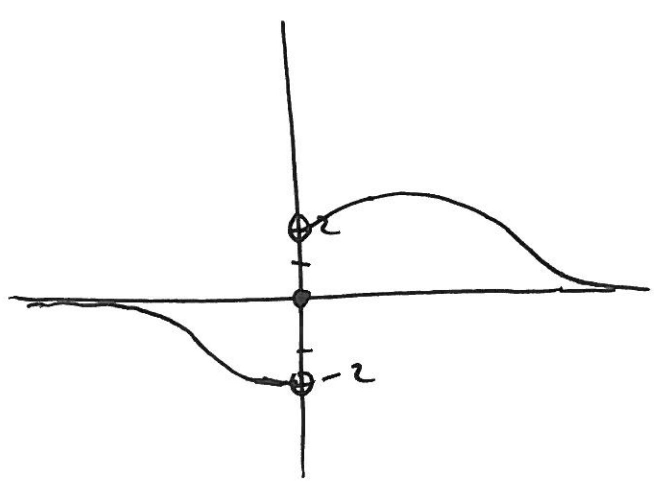
$\lim\limits_{x\to -1^+} \dfrac{2x}{x+1} = -\infty$ $\lim\limits_{x\to -1^-} \dfrac{2x}{x+1} = \infty$ vertical asymptote $x = -1$



(70) $f(0) = 0$, $\lim\limits_{x\to\pm\infty} f(x) = 0$, $\cancel{\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to -1^+} f(x) = \infty}$,

$\cancel{\lim\limits_{x\to 0^-} f(x) =}$ $\cancel{\lim\limits_{x\to -1^+} f(x) = \infty}$, $\cancel{\lim\limits_{x\to 1^+} f(x) = -\infty}$, $\cancel{\lim\limits_{x\to -1^-} f(x) = -\infty}$

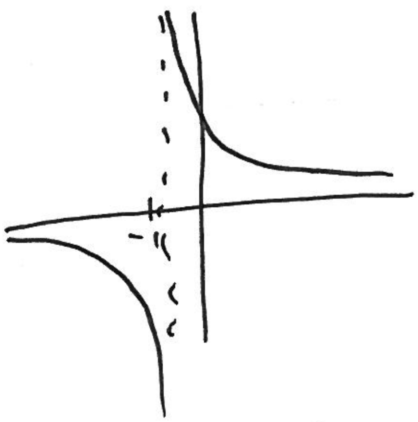
$\lim\limits_{x\to 0^+} f(x) = 2$, $\lim\limits_{x\to 0^-} f(x) = -2$



(64) $y = \frac{1}{x+1}$

$\lim\limits_{x \to \infty} \frac{1}{x+1} = 0 \Rightarrow$ horizontal asymptote $y = 0$

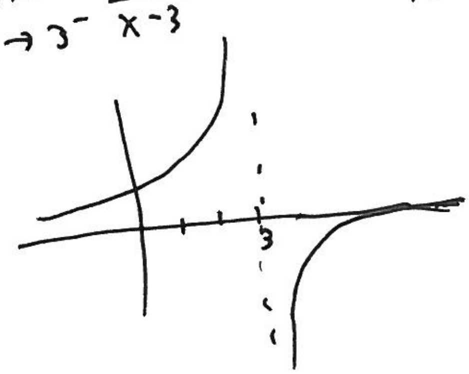
$\lim\limits_{x \to -1^+} \frac{1}{x+1} = \infty$, $\lim\limits_{x \to -1^-} \frac{1}{x+1} = -\infty \Rightarrow$ vertical asymptote $x = -1$



(66) $y = \frac{-3}{x-3}$

$\lim\limits_{x \to \infty} \frac{-3}{x-3} = 0 \Rightarrow$ horizontal asymptote $y = 0$

$\lim\limits_{x \to 3^-} \frac{-3}{x-3} = \infty$ $\lim\limits_{x \to 3^+} \frac{-3}{x-3} = \infty \Rightarrow$ vertical asymptote $x = 3$



(80) $\lim_{x \to \infty} \sqrt{x+9} - \sqrt{x+4}$ $(*)$

Note: $\sqrt{x+9} - \sqrt{x+4} = \dfrac{(\sqrt{x+9} - \sqrt{x+4})(\sqrt{x+9} + \sqrt{x+4})}{\sqrt{x+9} + \sqrt{x+4}}$

$$= \frac{(x+9)-(x+4)}{\sqrt{x+9}+\sqrt{x+4}}$$

$$= \frac{5}{\sqrt{x+9}+\sqrt{x+4}}$$

$(*) = \lim_{x \to \infty} \dfrac{5}{\sqrt{x+9}+\sqrt{x+4}} = 0.$

(90) Prove $\lim_{x \to 0} \dfrac{1}{|x|} = \infty$.

~~Given [illegible], we need to find B ≥ 0 such that~~

~~$0 < |x-2| < \delta$~~

Given $B > 0$ we need to find $\delta > 0$ such that

$0 < |x-0| < \delta \Rightarrow \dfrac{1}{|x|} > B.$

we want $\dfrac{1}{|x|} > B$ so $|x| < \dfrac{1}{B}$.

Choosing $\delta = \dfrac{1}{B}$ works.