2.5 (2, 4, 6, 8, 10, 14, 16, 18, 20, 26, 30, 32, 40, 44, 48, 53, 55)

(2) g fails to be continuous at 3, since $\lim_{x\to 3^-} g(x) \neq g(3)$.

(4) k fails to be continuous at 1 since $\lim_{x\to 1^-} k(x) \neq \lim_{x \to 1^+} k(x)$.

(6) a) Yes. $f(-1) = 0$.

b) yes. $\lim_{x\to -1^+} f(x) = 0$

c) Yes. See above.

d) Yes, by definition of continuity at an endpoint.

(10) change $f(1)$ to 2 since $\lim_{x\to 1} f(x) = 2$.

(18) Since $|x|+1$ is never 0, this function is continuous on $\mathbb{R}$.

(20) Recall $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. In particular, $\tan\theta$ is not continuous where $\cos\theta = 0$. Since $\cos\theta = 0$ when $\theta = \frac{\pi}{2} + k\pi$ for $k = \ldots, -1, 0, 1, \ldots$

$\cos\frac{\pi x}{2} = 0$ when $x = 2k+1$ for $k = \ldots, -1, 0, 1, \ldots$

Thus $\tan\left(\frac{\pi x}{2}\right)$ is continuous on $\mathbb{R} \setminus \{\ldots, -3, -1, 1, 3, \ldots\}$

odd integers.

(30) $$f(x) = \begin{cases} \dfrac{x^3-8}{x^2-4}, & x \neq 2,\ x \neq -2 \\ 3, & x = 2 \\ 4, & x = -2 \end{cases}$$

Note that

$$\frac{x^3-8}{x^2-4} = \frac{(x-2)(x^2+2x+4)}{(x+2)(x-2)}$$

It follows that $\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x^2+2x+4}{x+2} = \frac{4+4+4}{4} = 3 = f(2)$

so f is continuous at 2.

$\lim_{x\to -2} f(x) \neq f(-2)$ so f is not continuous at -2.

Every where else, f is defined by the rational function $\frac{x^3-8}{x^2-4}$, which is continuous on its domain.

Thus the function is continuous on $(-\infty, -2) \cup (-2, \infty)$.

(32) (*) $\lim_{t\to 0} \sin\left(\frac{\pi}{2} \cos(\tan t)\right)$. Note $\tan\theta$ is continuos at 0.
$\cos\theta$ is continuous.
$\sin\theta$ is continuous.

This is a composition of continuous functions so

$$(*) = \sin\left(\frac{\pi}{2}\cos(\tan(0))\right)$$
$$= \sin\left(\frac{\pi}{2}\cos(0)\right)$$
$$= \sin(\pi/2)$$
$$= 1$$

(40) $h(t) = \frac{t^2+3t-10}{t-2} = \frac{(t-2)(t+5)}{t-2}$

$\lim_{t\to 2} h(t) = \lim_{t\to 2} t+5 = 7$

define $h(2) = 7$.

(44) $g(x) = \begin{cases} x & , x < -2 \\ bx^2 & , x \geq -2 \end{cases}$

$\lim_{x \to -2^+} g(x) = (-2)^2 b = 4b$

$\lim_{x \to -2^-} g(x) = -2$

$4b = -2 \Rightarrow b = -1/2$

(48) $g(x) = \begin{cases} ax + 2b & x \leq 0 \\ x^2 + 3a - b & , 0 < x \leq 2 \\ 3x - 5 & , x > 2 \end{cases}$

$\lim_{x \to 0^-} g(x) = 2b$, $\lim_{x \to 0^+} g(x) = 3a - b$

$2b = 3a - b$

$3b = 3a$

$a = b.$

$\lim_{x \to 2^+} g(x) = 3(2) - 5 = 1$

$\lim_{x \to 2^-} g(x) = 4 + 3a - b$

$4 + 3a - b = 1$, $a = b$,

$4 + 2a = 1$

$2a = -3$

$a = -3/2$

$\therefore$ $a = -3/2 = b.$

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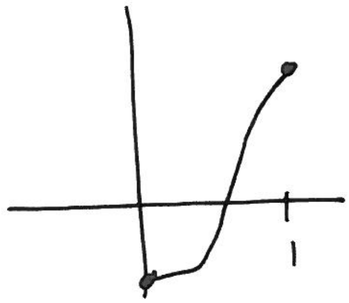

By Intermediate Value theorem. Choose $y_0 = 0$, which is between $f(0)$ and $f(1)$. IVT guarantees existence of c between 0 and 1 such that $f(c) = y_0$.

55 $f(x) = x^3 - 15x + 1$ is polynomial and so continuous.

$$f(-4) = (-4)^3 - 15(-4) + 1 = -3$$
$$f(4) = 4^3 - 15(4) + 1 = 5.$$

Choose $y_0 = 0$, which is between -3 and 5.
By IVT, there exist c between -4 and 4 such that
$f(c) = y_0$.