2.4 (2, 4, 6, 10, 18, 22, 30, 36, 44, 46)

② a) True
b) False
c) True
d) True
e) True
f) True
g) True
h) True
i) True
j) False , f is not defined to the left of -1
k) True , f is not defined to the right of 3

④ a) $\lim_{x \to 2^+} f(x) = 1$, $\lim_{x \to 2^-} f(x) = 1$, $f(2) = 2$

b) Yes. $\lim_{x \to 2} f(x) = 1$

c) $\lim_{x \to -1^-} f(x) = 4$, $\lim_{x \to -1^+} f(x) = 4$

d) Yes. $\lim_{x \to -1} f(x) = 4$

⑥ a) $\lim_{x \to 0^+} g(x) = 0$ using Squeeze Theorem.

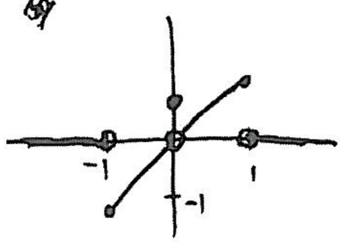
Note: $-\sqrt{x} \le g(x) \le \sqrt{x}$ for $x > 0$

$\lim_{x \to 0} -\sqrt{x} = \lim_{x \to 0} \sqrt{x} = 0$.

b) $\lim_{x \to 0^-} g(x)$ is undefined since g is not defined to the left of 0.

c) $\lim_{x \to 0} g(x)$ is undefined since g is not defined to the left of 0.

(10) a) domain is $\mathbb{R}$, range is $[-1, 1]$.



b) $\lim_{x \to c} f(x)$ exists for every c in $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

~~c) only The left-hand limit $\lim_{x \to c^-} f(x)$ exists for $c = -1$ and $c = 1$.~~

~~d) only the right-hand limit $\lim_{x \to c^+} f(x)$ exists for $c = -1$ and $c = 1$.~~

c) at all the points, the left hand limit exists. Sometimes it is different from the right hand limit, but that is ok.

d) Same as c).

(18) $$\lim_{x \to 1^+} \frac{\sqrt{2x}\,(x-1)}{|x-1|} \quad (*)$$

Notice that for $x > 1$, $x - 1 = |x - 1|$.

$$(*) = \lim_{x \to 1^+} \frac{\sqrt{2x}\,\cancel{(x-1)}}{\cancel{|x-1|}} = \sqrt{2}.$$

(22) $\lim_{t \to 0} \frac{\sin(kt)}{t} = \lim_{t \to 0} k \frac{\sin(kt)}{kt} = k \lim_{t \to 0} \frac{\sin(kt)}{kt}$ (*)

Let $\theta = kt$. as $t \to 0$, $\theta \to 0$.

(*) $= k \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = k$.

(30) $\lim_{x \to 0} \frac{x^2 - x + \sin x}{2x} = \frac{1}{2} \lim_{x \to 0} \left(x - 1 + \frac{\sin x}{x}\right)$

$= \frac{1}{2} \lim_{x \to 0} (x-1) + \frac{1}{2} \lim_{x \to 0} \frac{\sin x}{x}$

$= \frac{1}{2}(0 - 1 + 1) = 0$.

(36) $\lim_{x \to 0} \frac{\sin(5x)}{\sin(4x)} = \lim_{x \to 0} \frac{5}{4} \left(\frac{\sin(5x)}{5x}\right) \left(\frac{4x}{\sin(4x)}\right)$

$= \frac{5}{4} \lim_{x \to 0} \frac{\sin(5x)}{5x} \cdot \lim_{x \to 0} \frac{4x}{\sin 4x}$

$= \frac{5}{4}$

(44) Yes. If $\lim_{x \to c} f(x)$ exists, then $\lim_{x \to c^+} f(x)$ and $\lim_{x \to c^-} f(x)$ both exist and agree. ($\lim_{x \to c} f(x)$ is a 2 sided limit).

(46) If f is even, then $f(-x) = f(x)$ for all x.
It follows that if $\lim_{x \to 2^-} f(x) = 7$, then $\lim_{x \to -2^+} f(x) = 7$