

Name: _____ Academic Integrity Signature: _____

*I have abided by the UNCG Academic Integrity Policy.***Read all of the following information before starting the exam:**

- It is to your advantage to answer ALL of the 11 questions.
- It is your responsibility to make sure that you have all of the problems.
- There is no need to complete the test in order. The problems are independent.
- Correct numerical answers with insufficient justification may receive little or no credit.
- Clearly distinguish your final answer from your scratch work with a box or circle.
- *Budget your time!*
- If you have read all of these instructions, draw a happy face here.

Page:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

1. (10 points) Use the Intermediate Value Theorem to show that $f(x) = x^3 - x^2 - 1$ has a root in the interval $[1, 2]$.

2. (5 points) (Precise definition of limit) Let $f(x)$ be defined on an open interval containing x_0 , except possibly at x_0 itself. We say that the *limit of $f(x)$ as x approaches x_0 is L* , denoted $\lim_{x \rightarrow x_0} f(x) = L$, if

3. (5 points) Find a number $\delta > 0$ such that every number x in the interval $|x - 1| < \delta$ also satisfies $|(8x - 2) - 6| < \frac{1}{100}$.

4. Let $f(x) = \frac{x^2}{2x - 10}$.

(a) (5 points) Evaluate $\lim_{x \rightarrow 5^-} f(x)$ and $\lim_{x \rightarrow 5^+} f(x)$.

(b) (5 points) Does the graph $y = f(x)$ have a vertical asymptote? If it does, give the formula for the vertical asymptote. If not, explain why not.

5. Let $f(x) = \frac{3x^3 + 2x - 13}{7x^3 + 23x^2 + x - 1}$.

(a) (5 points) Evaluate $\lim_{x \rightarrow \infty} f(x)$.

(b) (5 points) Does the graph $y = f(x)$ have a horizontal asymptote? If it does, give the formula for the horizontal asymptote.

6. Evaluate the following limits

(a) (5 points) $\lim_{t \rightarrow -2} \frac{t + 2}{t^2 + 3t + 2}$

(b) (5 points) $\lim_{x \rightarrow 0} \frac{\sin(5x)}{3x}$

7. (10 points) Suppose f is a function such that $\lim_{x \rightarrow 1} f(x) = 2$. Suppose g is a function such that $\lim_{x \rightarrow 1} g(x) = 4$. Use Limit Laws to compute $\lim_{x \rightarrow 1} (4f(x) - \sqrt{g(x)})$.

8. (10 points) At what points is the function $f(x) = \frac{x+3}{x^2-3x-10}$ continuous?

9. (10 points) For what value of a is

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < -2, \\ 5ax & \text{if } x \geq -2 \end{cases}$$

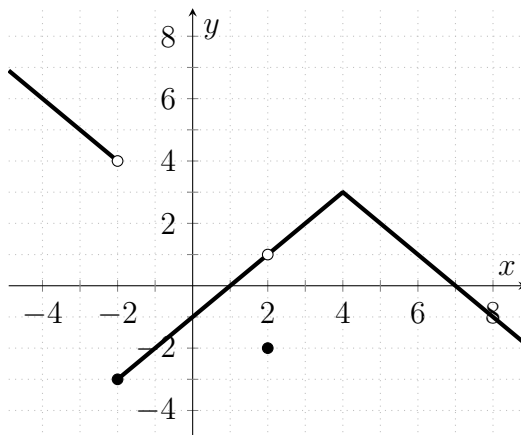
continuous at $x = -2$?

10. (10 points) Suppose f and g are continuous functions such that

$$\lim_{x \rightarrow 0} f(x) = 2, \quad f(7) = -1, \quad \lim_{x \rightarrow 0} g(x) = 7, \quad \text{and} \quad g(2) = 3.$$

Compute $\lim_{x \rightarrow 0} g(f(x))$ or explain what additional information is needed to compute the limit.

11. The graph of $y = f(x)$ is shown below. Compute the following or explain why it does not exist.



(a) (2 points) $\lim_{x \rightarrow -2^+} f(x)$

(b) (2 points) $\lim_{x \rightarrow -2^-} f(x)$

(c) (2 points) $f(-2)$

(d) (2 points) $\lim_{x \rightarrow 4} f(x)$

(e) (2 points) $\lim_{x \rightarrow 2} f(x)$