MAT 191 Test 1 September 11, 2012

Name: ____________________ Academic Integrity Signature: ____________________

I have abided by the UNCG Academic Integrity Policy.

Read all of the following information before starting the exam:

- It is to your advantage to answer ALL of the questions.
- It is your responsibility to make sure that you have all of the problems.
- There is no need to complete the test in order. The problems are independent.
- Correct numerical answers with insufficient justification may receive little or no credit.
- Clearly distinguish your final answer from your scratch work with a box or circle.
- *Budget your time!*
- If you have read all of these instructions, draw a happy face here.

Page:	2	3	4	5	Total
Points:	25	25	25	25	100
Score:					

1. (15 points) Let $f(x) = x^3 + 2x - 3$. Find the average rate of change of $f(x)$ over the interval $[1, 2]$.

Solution: We compute $f(2) = 2^3 + 2 \cdot 2 - 3 = 9$ and $f(1) = 1^3 + 2 \cdot 1 - 3 = 0$

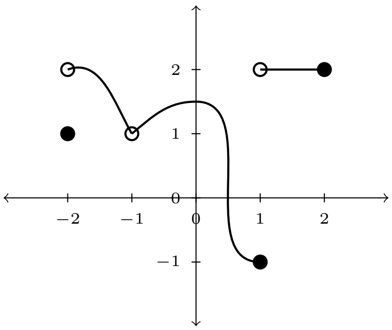
$$\begin{aligned} \frac{\Delta y}{\Delta x} &= \frac{f(2) - f(1)}{2 - 1} \\ &= \frac{9 - 0}{1} \\ &= 9. \end{aligned}$$

2. (10 points) Use the Intermediate Value Theorem to show that $x - \cos(x) = 0$ at some point x in the interval $[0, \frac{\pi}{2}]$.

Solution: First note that $f(x) = 2x - \cos(x)$ is continuous on $[0, \frac{\pi}{2}]$. (It is in fact continuous everywhere.) We have that $f(0) = 0 - \cos(0) = -1$ and $f(\frac{\pi}{2}) = \left(\frac{\pi}{2}\right)^2 - \cos(\frac{\pi}{2}) = \left(\frac{\pi}{2}\right)^2$. Since 0 is between -1 and $\left(\frac{\pi}{2}\right)^2$, the Intermediate Value Theorem guarantees the existence of c between 0 and $\frac{\pi}{2}$ such that $f(c) = 0$.

Points earned: ______ out of 25.

3. (15 points) Answer the following questions about $f(x)$. The entire graph of $f(x)$ is shown here. The axes that are shown are the x and y axes.



(a) What is $\lim_{x\to-2^+} f(x)$?

Solution: $\lim_{x\to-2^+} f(x) = 2$

(b) What is $\lim_{x\to-1^-} f(x)$?

Solution: $\lim_{x\to-1^-} f(x) = 1$

(c) What is $\lim_{x\to-1} f(x)$?

Solution: $\lim_{x\to-1} f(x) = 1$

(d) What is $\lim_{x\to1^+} f(x)$?

Solution: $\lim_{x\to-1} f(x) = 2$

(e) Does $\lim_{x\to1} f(x)$ exist? Explain.

Solution: No, since the left hand and right hand limits do not agree.

(f) Is $f(x)$ continuous at $x = -1$? Explain.

Solution: No, since the function is not defined at -1.

(g) Is $f(x)$ continuous at $x = 0$? Explain.

Solution: Yes. The left and right hand limits are both 1.5, which is equal to $f(0)$.

(h) Is $f(x)$ continuous at $x = 1$? Explain.

Solution: No, since the left and right hand limits do not agree.

4. (10 points) Find a number δ so that every number x in the interval $|x - \frac{1}{2}| < \delta$ also satisfies $|(6x-2)-1| < \frac{1}{10}$. Note: The definition of $\lim_{x\to\frac{1}{2}}(6x-2) = 1$ tells you that you can solve this problem.

 Points earned: ______ out of 25.

(Hint: Work backwards to find δ. You want $|(6x-2)-1| < \frac{1}{10}$, so figure out which x-values make this true.)

Solution: We have that

$$\begin{aligned}|(6x-2)-1| &= |6x-3| \\ &= 6|x-\frac{1}{2}| \\ &< \frac{1}{10}\end{aligned}$$

as long as $|x-\frac{1}{2}| < \frac{1}{60}$. Thus we can choose $\delta = \frac{1}{60}$.

5. (15 points) Let $f(x) = \dfrac{x^2}{2x-10}$.

   (a) Evaluate $\lim_{x\to 5^-} f(x)$.

   Solution: Note that if we tried to plug in $x = 5$, we would get "$\frac{25}{0}$". That means the limit is $+\infty$, $-\infty$, or does not exist. For values x near 5 but less than 5, x^2 is positive and $2x-10$ is negative. Thus $\dfrac{x^2}{2x-10}$ is negative for such values. It follows that

   $$\lim_{x\to 5^-} \frac{x^2}{2x-10} = -\infty.$$

   (b) Evaluate $\lim_{x\to 5^+} f(x)$.

   Solution: Note that if we tried to plug in $x = 5$, we would get "$\frac{25}{0}$". That means the limit is ∞, $-\infty$, or does not exist. For values x near 5 but greater than 5, x^2 is positive and $2x-10$ is positive. Thus $\dfrac{x^2}{2x-10}$ is positive for such values. It follows that

   $$\lim_{x\to 5^-} \frac{x^2}{2x-10} = \infty.$$

   (c) Does the graph $y = f(x)$ have a vertical asymptote? If it does, give the formula for the vertical asymptote.

   Solution: The only possible place this graph can have a vertical asymptote is where $2x-10=0$. That means $x=5$. The computations above show that $x=5$ is indeed a vertical asymptote.

6. (10 points) Let $f(x) = \dfrac{3x^2+2x-13}{7x^3+23x^2+x-1}$.

   (a) Evaluate $\lim_{x\to\infty} f(x)$.

Solution: Polynomials are dominated by their leading term. Hence

$$\begin{aligned}\lim_{x\to\infty}\frac{3x^2+2x-13}{7x^3+23x^2+x-1} &= \lim_{x\to\infty}\frac{3\cancel{x^2}}{7\cancel{x^3}^{1}} \\ &= \lim_{x\to\infty}\frac{3}{7x} \\ &= 0\end{aligned}$$

(b) Does the graph $y = f(x)$ have a horizontal asymptote? If it does, give the formula for the horizontal asymptote.

Solution: Yes. The horizontal asymptote (from the computation above) is $y = 0$.

7. (15 points) Evaluate the following limits

(a) $\lim_{y\to -1}\frac{y+1}{y^2+3y+2}$

Solution: Notice that if we plug in $y = -1$, we would get "$\frac{0}{0}$". That means we need to *do more work*. We compute

$$\begin{aligned}\lim_{y\to -1}\frac{y+1}{y^2+3y+2} &= \lim_{y\to -1}\frac{\cancel{(y+1)}}{\cancel{(y+1)}(y+2)} \\ &= \lim_{y\to -1}\frac{1}{y+2} \\ &= 1.\end{aligned}$$

(b) $\lim_{x\to 0^+}\frac{x+2-\sqrt{x}}{\cos(x)}$

Solution: Since all the functions involved are continuous at 0, we can compute the limit by just plugging in $x = 0$. Thus the limit is $\frac{0+2-\sqrt{0}}{\cos(0)} = \frac{2}{1} = 2$.

8. (10 points) For what value of a is

$$f(x) = \begin{cases} x^2+3 & \text{if } x < 3, \\ 2ax & \text{if } x \geq 3 \end{cases}$$

continuous at $x = 3$?

Solution: We have that f is continuous at $x = 3$ if $\lim_{x\to 3} f(x) = f(3)$. We compute $\lim_{x\to 3^-} f(x) = 3^2 + 3 = 12$ and $\lim_{x\to 3^+} f(x) = 2\cdot a\cdot 3 = 6a$. It follows that to arrange that f is continuous at $x = 3$, we must have $12 = 6a$. Thus choosing $a = 2$ will make f continuous at $x = 3$.