Mini-Lecture 1.1
The Distance and Midpoint Formulas

<u>Learning Objectives</u>:

1. Use the Distance Formula
2. Use the Midpoint Formula

<u>Examples</u>:

1. Find the distance between the points (-3,7) and (4,10).

2. Determine whether the triangle formed by points A=(-2,2), B=(2,-1), and C=(5,4) is a right triangle.

3. Find the midpoint of the line segment joining the points P_1=(6,-3) and P_2=(4,2).

<u>Teaching Notes</u>:

- Go over the terms used in introducing the rectangular coordinate system.
- Tell them the distance formula will be used in several applications later in the course.
- Students don't have much trouble with the distance formula, but they will sometimes reverse the order of the coordinates or will make careless arithmetic mistakes such as using subtraction instead of addition.
- The midpoint formula is also fairly easy for them, but students will sometimes have trouble if the coordinates include fractions.

<u>Answers</u>:

1. $\sqrt{58}$ 2. $No: |AB|^2 = 13,\ |BC|^2 = 34,\ |AC|^2 = 53.$ 3. $\left(5, -\frac{1}{2}\right)$

Mini-Lecture 1.2
Graphs of Equations in Two Variables; Intercepts; Symmetry

Learning Objectives:

1. Graph equations by plotting points
2. Find intercepts from a graph
3. Test an equation for symmetry with respect to the x-axis, the y-axis, and the origin
4. Know how to graph key equations

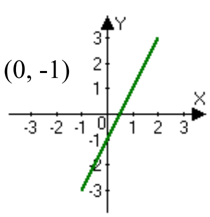
Examples:

1. Determine whether the points (0,3), (-2,0), and (2,7) are on the graph of the equation $y = x^3 - 2x + 3$.
2. Find the intercepts of the equation $y = 2x - 1$ by plotting points.
3. List the intercepts and test for symmetry for each equation.
   $(a)\ y^2 - x - 4 = 0$ $(b)\ y = \dfrac{x}{x^2 - 4}$

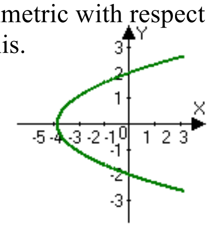
Teaching Notes:

- When graphing by plotting points, be sure to emphasize that the *y*-coordinate is determined by the value of *x*. This will help establish the function concept later.
- Emphasize how to find intercepts algebraically by setting *x=0* to find the *y*-intercept(s) and then *y=0* to find the *x*-intercept(s).
- Symmetry can be seen and identified, but students will often have trouble testing for symmetry algebraically. They will make a lot of sign errors, so that needs to be reinforced.
- Emphasize the graphing of the key functions. It is important that they know the basic shapes of these graphs when this topic is revisited later in the course.

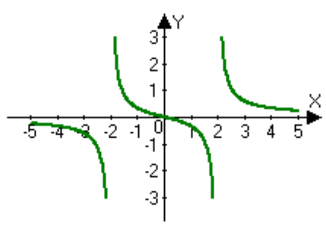
Answers: **1.** Yes, No, Yes **2.** (1/2,0), (0, -1)



3. (a) (0,2),(0,-2),(-4,0)
Symmetric with respect to the *x*-axis.



3. (b) (0,0) Symmetric with respect to the origin.



Mini-Lecture 1.3
Lines

Learning Objectives:

1. Calculate and interpret the slope of a line
2. Graph lines given a point and a slope
3. Find the equation of a vertical line
4. Use the point-slope form of a line; identify horizontal lines
5. Find the equation of a line, given two points
6. Write the equation of a line in slope-intercept form
7. Identify slope and y-intercept of a line from its equation
8. Graph lines written in general form using intercepts
9. Find equations of parallel lines
10. Find equations of perpendicular lines

Examples:

1. Determine the slope of the line containing the points (-5,4) and (0,7).
2. Graph the line containing the point $(2,4)$ with slope $m = \frac{-2}{3}$.
3. Write an equation of the line satisfying the given conditions:
   (a) Slope = $\frac{3}{4}$, containing the point (-2,4). (b) Containing the points (4,2) and (3,-4).
   (c) x-intercept = 3, y-intercept = -2. (d) Vertical line containing (5,-1).
   (e) Parallel to the line $3x - 4y = 5$ and containing the point (3,-6).
4. Find the slope and y-intercept of the line $4x - 6y = -3$.
5. Find the intercepts and graph the line $-2x + y = 4$.

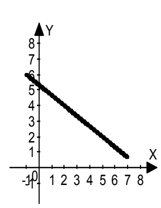
Teaching Notes:

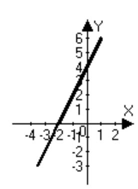
- This material is not usually hard for the students.
- When finding the slope, make sure they don't reverse the x- and y-values.
- They should learn the various forms for the equation of a line and be comfortable solving standard form for y.
- Simplifying the equations should be emphasized.

Answers: 1. $m = \frac{7-4}{0-(-5)} = \frac{3}{5}$ 3. (a) $y = \frac{3}{4}x + \frac{11}{2}$

3. (b) $y = 6x - 22$ (c) $y = \frac{2}{3}x - 2$ (d) $x = 5$ (e) $y = \frac{3}{4}x - \frac{33}{4}$

4. Slope $= \frac{2}{3}$; y-intercept $= \frac{1}{2}$ 5. x-intercept = -2, y-intercept = 4





Mini-Lecture 1.4
Circles

Learning Objectives:

1. Write the standard form of the equation of a circle
2. Graph a circle
3. Work with the general form of the equation of a circle

Examples:

1. Write the standard form and general form of the equation of each circle with radius r and center (h,k). Graph each circle.

   $(a)\ r=3;\ (h,k)=(-2,3).\qquad (b)\ r=\frac{2}{3};\ (h,k)=(0,0).$

2. Find the center (h,k) and radius r of each circle.

   $(a)\ 2(x-2)^2+2(y+3)^2=8\qquad (b)\ x^2+y^2-6x+2y+4=0$

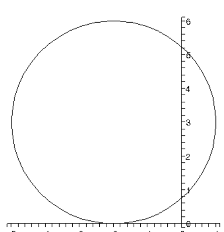
3. Find the general form of the equation of each circle.
   (a) Center (2,-3) and containing the point (0,4).
   (b) Endpoints of a diameter at (6,10) and (-4,-4).

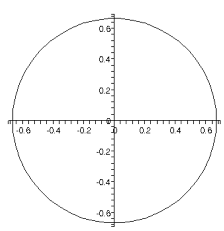
Teaching Notes:

- Show the difference in the equations of circles with centers at the origin and those with centers elsewhere.
- It is not necessary to memorize the general form of the equation.
- Students will need review on completing the square (in order to put the general form into standard form).

Answers:

1. $(a)\ (x+2)^2+(y-3)^2=9;\ x^2+y^2+4x-6y+4=0.\quad (b)\ x^2+y^2=\frac{4}{9};\ x^2+y^2-\frac{4}{9}=0.$





2. $(a)\ c=(2,-3);\ r=2.\qquad (b)\ c=(3,-1);\ r=\sqrt{6}.$
3. $(a)\ (x-2)^2+(y+3)^2=53.\qquad (b)\ (x-1)^2+(y-3)^2=74.$

Mini-Lecture 2.1
Functions

Learning Objectives:

1. Determine whether a relation represents a function
2. Find the value of a function
3. Find the domain of a function defined by an equation
4. Form the sum, difference, product, and quotient of two functions

Examples:

1. Determine whether the equation defines y as function of x.

   $(a)\ y = x^2 - 2x \quad (b)\ y^2 = 3x - 4 \quad (c)\ 5x + 7y = 10 \quad (d)\ y = \dfrac{2}{x-3}$

2. For $f(x) = -x^2 + 2x - 3$ find $(a)\ f(0) \quad (b)\ f(-1) \quad (c)\ f(3)$
3. Find the domain of each function.

   $(a)\ f(x) = 2x + 3 \quad (b)\ f(x) = \dfrac{2}{x^2} \quad (c)\ f(x) = \dfrac{2x}{x^2+1} \quad (d)\ f(x) = \dfrac{5}{\sqrt{x+4}}$

4. For $f(x) = 2x - 3$ and $g(x) = 2x^2$, find

   $(a)\ (f+g)(x) \quad (b)\ (f-g)(x) \quad (c)(f \cdot g)(2) \quad (d)\left(\dfrac{f}{g}\right)(3)$

Teaching Notes:

- This is a critical section. If the students do not understand the concept of a function, they will struggle throughout the course.
- Spend time on the correspondence aspect of a function. You may use the birthday example. Every student has only one birthday, but other students can have that same birthday. Emphasize that no student has two birthdays.
- Demonstrate the difference between a relation and a function. A circle and a line are good geometric examples of this. A function is a special type of relation.
- The input – output machine in figure 10 is a good one to use extensively. Just keep emphasizing "input=domain, output=range".
- If time permits, introduce the difference quotient as a precursor to calculus limits.

Answers: 1. (a) yes (b) no (c) yes (d) yes 2. (a) -3 (b) -6 (c) -6

3. $(a)\ (-\infty,\infty) \quad (b)\ (-\infty,0)\cup(0,\infty) \quad (c)(-\infty,\infty) \quad (d)\ (-4,\infty)$

4. $(a)\ 2x^2 + 2x - 3 \quad (b)\ -2x^2 + 2x - 3 \quad (c)\ 8 \quad (d)\ \dfrac{1}{6}$

Mini-Lecture 2.2
The Graph of a Function

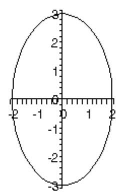
Learning Objectives:

1. Identify the graph of a function
2. Obtain information from or about the graph of a function

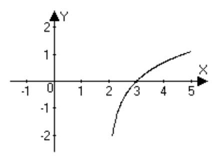
Examples:

1. Determine whether the graph is that of a function. If it is, then use the graph to find the domain, range, any intercepts, and symmetry with respect to the *x*-axis, the *y*-axis, or the origin.

(a)



(b)



2. For $f(x) = \dfrac{2x}{x-2}$ answer the following questions.
   (a) Is the point (3,6) on the graph of *f*?
   (b) For $x = -2$, what is $f(x)$? What are the coordinates of that point on the graph $y=f(x)$?
   (c) If $f(x) = 3$, what is x?
   (d) What is the domain of *f*?
   (e) List any intercepts and zeros of *f*.

Teaching Notes:

- The vertical-line test is a useful tool if a student has a graph to analyze.
- Draw a lot of different graphs, and use the vertical-line test. This will help establish the concept of a function.
- Spend a good amount of time having the students read information from graphs. This is often something they have difficulty with. This will also help them later when they are learning about increasing and decreasing functions.

Answers:

1. (a) No (b) Function; Domain= $(2,\infty)$, Range = $(-\infty,\infty)$, *x*-int=3, no symmetry.

2. (a) Yes (b) 1; $(-2,1)$ (c) $x = 6$ (d) $(-\infty,2)\cup(2,\infty)$

   (e) *x*-int=0; *y*-int=0; zero = 0.

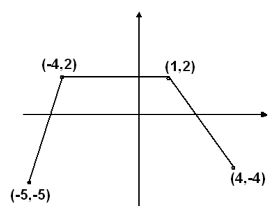
Mini-Lecture 2.3
Properties of Functions

Learning Objectives:

1. Determine even and odd functions from a graph
2. Identify even and odd functions from the equation
3. Use a graph to determine where a function is increasing, decreasing, or constant
4. Use a graph to locate local maxima and local minima
5. Use a graph to locate the absolute maximum and the absolute minimum
6. Use a graphing utility to approximate local maxima and local minima and to determine where a function is increasing or decreasing
7. Find the average rate of change of a function

Examples:

1. For the graph below,
   (a) State the intervals where the function is increasing, decreasing, or constant.
   (b) State the domain and range.
   (c) State whether the graph is odd, even or neither.
   (d) Locate the maxima and minima.



2. Determine algebraically whether the function $f(x) = x^3 - 2x + 1$ is odd, even, or neither.
3. Find the average rate of change of $f(x) = -x^3 + 3x^2$ from x=-1 to x=4.

Teaching Notes:

- Graphically determining even and odd will not present a problem, but determining this algebraically can be difficult for students. Show the graph of a function and give its algebraic definition at the same time. This will help reinforce this concept.
- Determining increasing, decreasing, or constant is done fairly easily by just drawing some graphs and going over the properties. This is fairly intuitive. The main difficulty the students will have is showing the proper intervals. They will often want to use the *y*-values and not the *x*-values in their intervals.
- Students will usually give a point and not a value for a max or min. They need to understand that a max or min is a value and not an ordered pair.

Answers:

1. (a) Increasing on $(-5,-4)$; Decreasing on $(1,4)$; Continuous on $(-4,1)$
   (b) Domain = $[-5,4]$; Range = $[-5,2]$.   (c) Not odd or even.
   (d) Local Maximum = 2, Local Minimum = -5.
2. Neither   3. -4

Mini-Lecture 2.4
Library of Functions; Piecewise-defined Functions

<u>Learning Objectives</u>:

1. Graph the functions listed in the library of functions
2. Graph piecewise-defined functions

<u>Examples</u>: There are no variations from the library of functions in the exercises; this will be done in later sections. Therefore, these examples are of piece-wise functions only, but all of the library of functions are included in them.

1. Sketch the graph of each function.

$$(a)\ f(x)=\begin{cases}2x-1 & \text{if } x>2,\\ 2-x & \text{if } x\le 2.\end{cases} \qquad (b)\ f(x)=\begin{cases}\dfrac{1}{x} & \text{if } x<0,\\ \sqrt{x} & \text{if } x\ge 0.\end{cases}$$

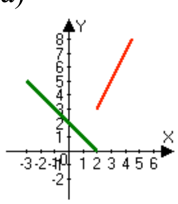
$$(c)\ f(x)=\begin{cases}x^3 & \text{if } x<1,\\ |x| & \text{if } x\ge 1.\end{cases} \qquad (d)\ f(x)=\begin{cases}x^2 & x<0,\\ 1 & x=0,\\ \sqrt[3]{x} & x>0.\end{cases}$$

<u>Teaching Notes</u>:

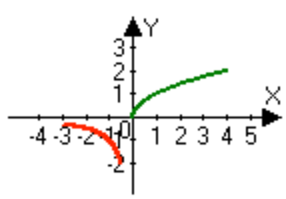
- It is a good idea to have the students memorize the graphs of the functions listed in the library of functions. Plotting points may help them at the beginning, but the graphs should be committed to memory.
- Students often have trouble graphing piecewise functions. If you can show the different parts in different colors, this can help them visualize the way the function is divided.

Answers:

1. (a)



(b)



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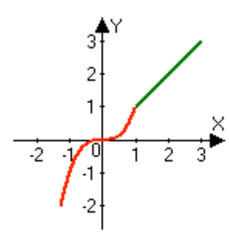

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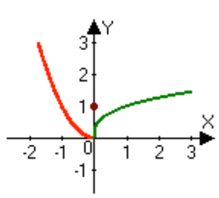


Mini-Lecture 2.5
Graphing Techniques: Transformations

Learning Objectives:

1. Graph functions using vertical and horizontal shifts
2. Graph functions using compressions and stretches
3. Graph functions using reflections about the *x*-axis and the *y*-axis

Examples:

1. Sketch the graph of each function.

$(a)\ f(x) = x^2 - 2$ $\quad (b)\ f(x) = x^3 + 3$ $\quad (c)\ f(x) = \sqrt{x+5}$ $\quad (d)\ f(x) = |x-2|$

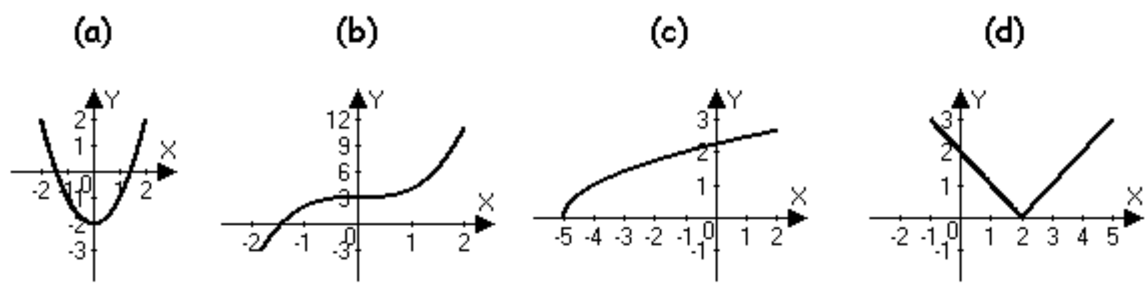
$(e)\ f(x) = 2x^2$ $\quad (f)\ f(x) = \frac{1}{2}x^3$ $\quad (g)\ f(x) = -\dfrac{1}{x}$ $\quad (h)\ f(x) = \sqrt{3-x}$

$(i)\ f(x) = (x-1)^2 + 2$ $\quad (j)\ f(x) = -\sqrt{2-x} + 1$

Teaching Notes:

- If you can use a graphing calculator or computer projection with Mathematica or Maple, it is easy to show the transformations with multiple examples. The more examples the better.
- Using a table and plotting points can be helpful, but is too time-consuming to use on most graphs.
- Take just one of the functions, such as $f(x) = x^2$, and do all of the transformations. Then do the other functions.
- Problems like #63-66 are excellent ways to see if students understand the concept.

Answers:



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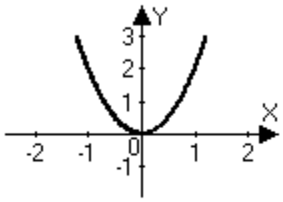
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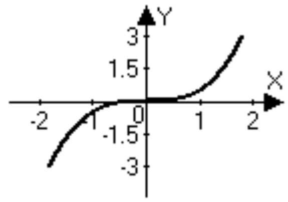
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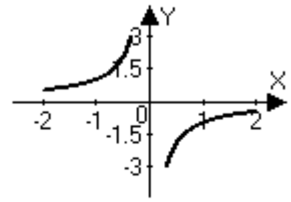
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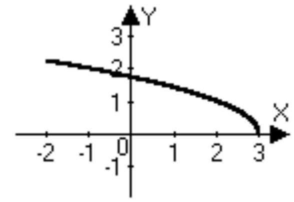
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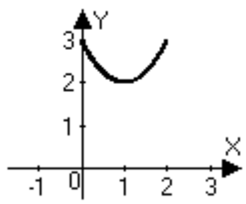
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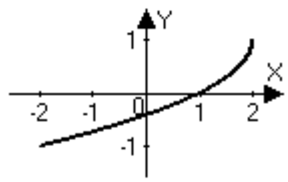
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Mini-Lecture 2.6
Mathematical Models: Building Functions

Learning Objectives:

1. Build and analyze functions

Examples:

1. Two cars are approaching an intersection. One is 1 mile south of the intersection and is moving at a constant speed of 40 mph. At the same time, the other car is 2 miles east of the intersection and is moving at a constant speed of 10 mph.
   (a) Express the distance *d* between the cars as a function of time *t*.
   (b) For what value of *t* is *d* smallest?

2. A rectangle has one corner on the graph of $y = 9 - x^2$, another at the origin, a third on the positive *y*-axis, and the fourth on the positive *x*-axis.
   (a) Express the area *A* as a function of *x*.
   (b) For what value of *x* is *A* the largest?
   (c) What is the domain of *A*?

3. Let $P = (x, y)$ be a point on the graph of $y = x^2 - 25$.
   (a) Express the distance *d* from *P* to the point $(1, 0)$ as a function of *x*.
   (b) What is *d* if *x*=2?

Teaching Notes:

- Students do not like word problems, so it is essential that your examples be creative.
- Try to use charts when possible.

Answers:

1. (a) $d = \sqrt{1700t^2 - 120t + 5}$ (b) $t \approx 0.035$ hours

2. (a) $A(x) = 9x - x^3$ (b) $x = \sqrt{3}$ (c) (0,3)

3. (a) $d = \sqrt{x^4 - 49x^2 - 2x + 626}$ (b) 21

Mini-Lecture 3.1
Linear Functions and Their Properties

Learning Objectives:
1. Graph linear functions
2. Use average rate of change to identify linear functions
3. Determine whether a linear function is increasing, decreasing, or constant
4. Build linear models from verbal description

Examples:

1. For each function, (i) determine the slope and *y*-intercept; (ii) graph the function using slope and *y*-intercept; (iii) determine the average rate of change; and (iv) determine whether the function is increasing, decreasing, or constant.

$(a)\ f(x) = 3x + 5.$ $(b)\ f(x) = -4x + 2.$ $(c)\ f(x) = 5.$

2. Suppose $f(x) = 3x + 6$ and $g(x) = -x + 4$.

(a) Solve $f(x) = 0$. (b) Solve $f(x) \geq 0$.

(c) Solve $f(x) = g(x)$. (d) Solve $f(x) \leq g(x)$.

3. The cost, *C*, in dollars of a cellular phone plan is given by the function $C(x) = 0.30x + 7$, where *x* is the number of minutes used.

(a) What is the cost of the plan if you talk for 150 minutes?
(b) If the bill is $220, how many minutes were used?
(c) What is the maximum number of minutes that can be used for $120?

Teaching Notes:
- The delta notation may take them a bit to understand.
- The applications are not difficult and should be interesting. Emphasize these.

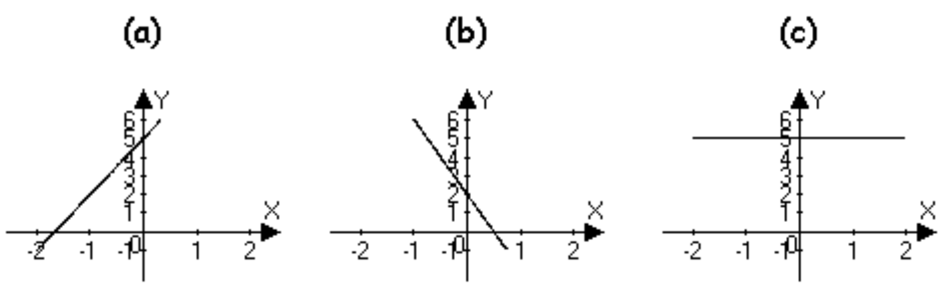
Answers: (Graphs are below.)

1. (a) (i) Slope = 3, *y*-intercept = 5; (iii) 3; (iv) increasing.
   (b) (i) Slope = -4, *y*-intercept = 2; (iii) -4; (iv) decreasing.
   (c) (i) Slope = 0, *y*-intercept = 5; (iii) 0; (iv) constant.

2. $(a)\ x = -2$ $(b)(-2, \infty)$ $(c) x = -\frac{1}{2}$ $(d)\left(-\infty, -\frac{1}{2}\right]$

3. (a) $52 (b) 710 (c) 376

1 .



Mini-Lecture 3.2
Linear Models: Building Linear Functions from Data

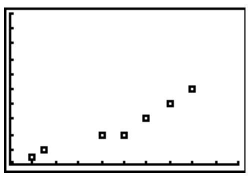
Learning Objectives:

1. Draw and interpret scatter diagrams
2. Distinguish between linear and nonlinear relations
3. Use a graphing utility to find the line of best fit

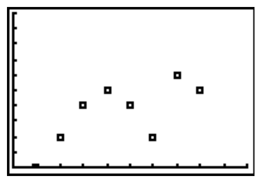
Examples:

1. Examine the scatter diagram and determine whether the relation is linear or nonlinear.

(a)



(b)



2. For the data below, draw a scatter diagram. Select two points from the diagram, and find the equation of the line containing the two points selected. Graph the line found on the scatter diagram.

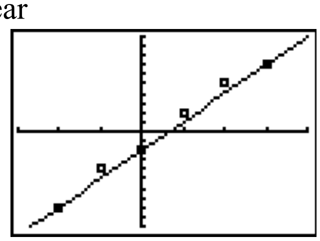
x	-2	-1	0	1	2	3
y	-8	-4	-2	2	5	7

Teaching Notes:

- Students will enjoy this, but if they don't have a graphing calculator you need to keep the examples simple.
- This is a good chance to make the students aware of real world uses of mathematics.

Answers:

1. (a) linear (b) nonlinear
2. $y = 3x - 2$



Mini-Lecture 3.3
Properties of Quadratic Functions

Learning Objectives:

1. Graph a quadratic function using transformations
2. Identify the vertex and axis of symmetry of a quadratic function
3. Graph a quadratic function using its vertex, axis, and intercepts
4. Find a quadratic function given its vertex and one other point
5. Find the maximum or minimum value of a quadratic function

Examples:

1. Graph each function by using transformations on the function $f(x)=x^2$.

$(a)\, f(x)=2(x-2)^2-2 \qquad (b)\; f(x)=-3(x+1)^2+3$

2. Find the vertex, axis of symmetry, and intercepts, then graph the function. State the domain and range, where the function is increasing, and where it is decreasing.

$(a)\, f(x)=2x^2-3x-2 \qquad (b)\; f(x)=-x^2-4x$

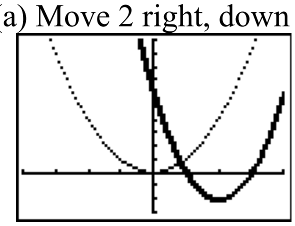
3. Find the quadratic function whose vertex is at (2,5) and passes through (3,2).
4. Determine the value of the maximum or the minimum without graphing.

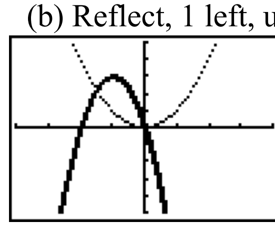
$(a)\, f(x)=3x^2-24x+53 \qquad (b)\; f(x)=-2x^2-12x-24$

Teaching Notes:

- Initially, students will get confused with horizontal transformations, but vertical transformations don't cause too much difficulty.
- It is important that they learn to put the function in the form $f(x)=a(x-h)^2+k$ in order to identify the vertex and to graph the function. It is also a good idea to teach them to use this form to find any x-intercepts by solving $a(x-h)^2+k=0$, especially when the function does not factor.
- It is important that they use the form $f(x)=ax^2+bx+c$ to find the y-intercept. Otherwise they will think that the k value is the y-intercept.
- Students will often state the vertex as the maximum or minimum instead of the y-value. Emphasize that the maximum or minimum is a value, not a coordinate.

Answers: 1.(a) Move 2 right, down 2, stretch of 2 (b) Reflect, 1 left, up 3, stretch of 3

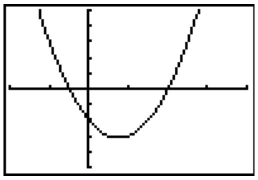




2. (a) Vertex = (3/4,-25/8); Axis of symmetry: x=3/4; *x*-intercepts -1/2, 2; *y*-intercept = -2
Domain = $(-\infty, \infty)$, Range = $[-25/8, \infty)$, Decreasing $(-\infty, 3/4)$, Increasing $(3/4, \infty)$

(b) Vertex = (-2,4); Axis of symmetry: x=-2; *x*-intercepts -4, 0; *y*-intercept = 0
Domain = $(-\infty, \infty)$, Range = $(-\infty, 4]$, Increasing $(-\infty, -2)$, Decreasing $(-2, \infty)$

(a)



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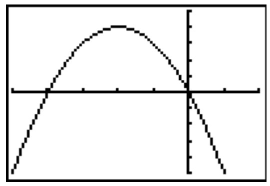


3. $f(x) = -3(x-2)^2 + 5$

4. (a) Minimum is 5 (b) Maximum is -6

Mini-Lecture 3.4
Build Quadratic Models from Verbal Descriptions and Data

Learning Objectives:

1. Build quadratic models from verbal descriptions
2. Build quadratic models from data

Examples:

1. An object is propelled straight upward from a height of 6 feet with an initial velocity of 32 feet per second. The height at any time *t* is given by $s(t) = -16t^2 + 32t + 6$ where $s(t)$ is measured in feet and *t* in seconds. Find the maximum height attained by the object.

2. A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals. What dimensions will produce a maximum enclosed area?

3. The revenue function for a new plasma television is given by $R(p) = 900p - 0.1p^2$. What price, *p*, should be charged to maximize revenue? What is the maximum revenue?

Teaching Notes:

- Students do not like application problems. You need to make the examples relevant.
- Emphasize the need to study problems of different types and to see a pattern in the way they are set up.
- Show the students that the equations are not difficult to solve, once the model is established.

Answers:

1. 22 feet

2. $50\,\text{ft} \times 33\frac{1}{3}\,\text{ft}$

3. \$4500; \$2,025,000

Mini-Lecture 3.5
Inequalities Involving Quadratic Functions

Learning Objectives:

1. Solve inequalities involving a quadratic function

Examples:

1. Solve each inequality.

$(a)\ x^2 - x - 12 \le 0$

$(b)\ -2x^2 > -11x + 15$

$(c)\ 3x^2 + 6x > 45$

Teaching Notes:

- A problem that students have with inequalities is the notation. Make sure they understand how to properly express the answer in interval notation.

Answers:

1. $(a)\ [-3, 4]$ $\quad (b)\ \left(\frac{5}{2}, 3\right)$ $\quad (c)\ (-\infty, -5) \cup (3, \infty)$

Mini-Lecture 4.1
Polynomial Functions and Models

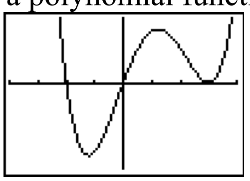
Learning Objectives:

1. Identify polynomial functions and their degree
2. Graph polynomial functions using transformations
3. Identify the real zeros of a polynomial function and their multiplicity
4. Analyze the graph of a polynomial function
5. Build cubic models from data

Examples:

1. Determine which functions are polynomials and state the degree. If not, state why.
   $(a)\ f(x)=5x^3-3x+4 \quad (b)\ f(x)=\sqrt{3}x^2 \quad (c)\ f(x)=\dfrac{3+x}{x-3} \quad (d)\ f(x)=\sqrt{x-3}$
2. Form a polynomial whose degree and zeros are given. Don't expand.
   (a) Degree 3; zeros: -2,0,4 (b) Degree 4; zeros: -2, multiplicity 3; 1, multiplicity 1

3. Find a polynomial function that could form the graph shown below.



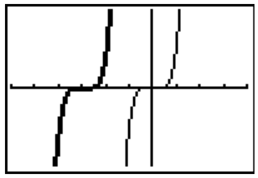
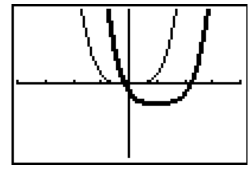
4. Use transformations of $y=x^4$ or $y=x^5$ to graph each function.
   $(a)\ f(x)=(x+3)^5 \quad (b)\ f(x)=x^5+3 \quad (c)\ f(x)=(x-1)^4-2$
5. Sketch the graph of each function by using end-behavior and multiplicity of zeros.
   $(a)\ f(x)=x^3(x-1)(x+3) \quad (b)\ f(x)=(x+2)^2(x-2)(x+4)$

Teaching Notes:

- It is essential that students understand what constitutes a polynomial function.
- Emphasize the behavior of the functions at their zeros and the end behavior
- Giving students a generic graph using a,b,c instead of numerical values can be helpful in getting them to master the concepts.

Answers:

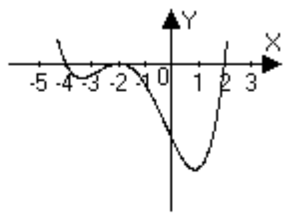
1. (a) Polynomial, degree 3 (b) Polynomial, degree 2 (c) Not a polynomial because the variable is in the denominator (d) Not a polynomial because of the radical
2. $(a)\ f(x)=x(x+2)(x-4) \quad (b)\ f(x)=(x+2)^3(x-1)$
3. $f(x)=x(x+2)(x-3)^2$

4. (a)  (b)  (c) 

5.

(a)

(b)



Mini-Lecture 4.2
Properties of Rational Functions

Learning Objectives:
1. Find the domain of a rational function
2. Find the vertical asymptotes of a rational function
3. Find the horizontal or oblique asymptotes of a rational function

Examples:

1. Find the domain of each rational function.

$(a)\ f(x)=\dfrac{2-x}{x+2}\quad (b)\ f(x)=\dfrac{3x^2}{x^2-9}\quad (c)\ f(x)=\dfrac{x-1}{x^2+1}\quad (d)\ f(x)=\dfrac{x}{x^2-4x-5}$

2. Identify vertical asymptotes, horizontal asymptotes, and oblique asymptotes.

$(a)\ f(x)=\dfrac{4x+1}{3-x}\quad (b)\ f(x)=\dfrac{2x^3+3x}{x^2-1}\quad (c)\ f(x)=\dfrac{x^4+1}{x^2-9}\quad (d)\ f(x)=\dfrac{2x}{x^2+1}$

3. Graph each function.

$(a)\ f(x)=\dfrac{1}{x}-2\qquad (b)\ f(x)=\dfrac{2}{(x-2)^2}+1$

Teaching Notes:
- Students will often remove from the domain values of x that make the numerator 0. For example, for $f(x)=\dfrac{x-3}{x^2-4}$, they will erroneously state the domain as $\{x|x\neq -2, x\neq 2, x\neq 3\}$.
- Emphasize analyzing the function to determine asymptotes so that the students can learn to determine asymptotes quickly.

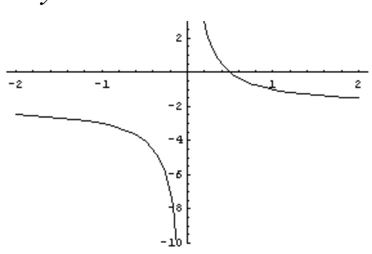
Answers:

1. $(a)\ \{x|x\neq -2\}$ $(b)\ \{x|x\neq -3, x\neq 3\}$ (c) All Real Numbers (d) $\{x|x\neq -1, x\neq 5\}$

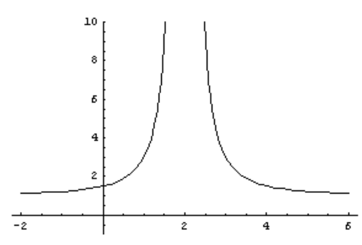
2. (a) V.A. $x=3$, H.A. $y=-4$ (b) Oblique $y=2x$, V.A. $x=1, x=-1$ (c) V.A. $x=3, x=-3$

   (d) H.A. $y=0$

3. (a)



(b)



Mini-Lecture 4.3
The Graph of a Rational Function

Learning Objectives:

1. Analyze the graph of a rational function
2. Solve applied problems involving rational functions

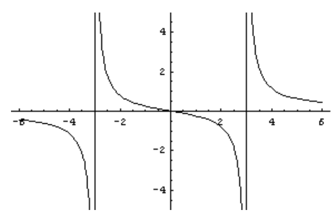
Examples:

1. Analyze the graph of $f(x) = \dfrac{2x}{x^2 - 9}$ with the 7-step process used in Example 2.
2. A company that produces snowmobiles has a cost function, $C(x) = 3500x + 150{,}000$.
   (a) Find the average cost function. (b) What is the average cost of producing 100 snowmobiles?

Teaching Notes:

- Students usually do not enjoy application problems, but these problems have a lot of interesting applications. Try to make it relevant for the students by using current data and functions.
- If the students have a graphing calculator, have them try to do the graphs and use the calculator to check their work. They need to grasp the concepts without too much dependency on a graphing calculator to help.
- Emphasize analyzing the functions without looking at the graph. Look at the graphs after the analysis is done, to check your work.

Answers:



1. Step 1: Domain is $\{x \mid x \neq 3, x \neq -3\}$, $f(0) = 0$.
   Step 2: Reduced
   Step 3: VA are x = 3 and x = -3
   Step 4: HA is y = 0, intersects at the point (0,0)
   Step 5: On the interval $(-\infty, -3)$, the function is below the *x*-axis.
   On the interval $(-3, 0)$, the function is above the *x*-axis.
   On the interval $(0, 3)$, the function is below the *x*-axis.
   On the interval $(3, \infty)$, the function is above the *x*-axis.
   Step 6: As $x \to -\infty,\ f(x) \to 0;\ x \to -3^-, f(x) \to -\infty;\ x \to -3^+, f(x) \to \infty$
   $x \to \infty,\ f(x) \to 0;\ x \to 3^-, f(x) \to -\infty;\ x \to 3^+, f(x) \to \infty$
2. (a) $\overline{C}(x) = 3500 + \dfrac{150{,}000}{x}$ (b) \$5000

Mini-Lecture 4.4
Polynomial and Rational Inequalities

Learning Objectives:

1. Solve polynomial inequalities
2. Solve rational inequalities

Examples:

1. Solve each polynomial inequality.

$(a)\ (x+3)^2(x-4)>0$ $\qquad (b)\ (x+1)(x-5)(x+3)\le 0$

$(c)\ x^6>16x^4$ $\qquad (d)\ x^3+2x^2-8x\le 0$

2. Solve each rational inequality.

$(a)\ \dfrac{(x+3)^2}{x^2-4}\ge 0$ $\qquad (b)\ \dfrac{2x-1}{x+4}\le 2$

$(c)\ \dfrac{x^2(x+3)(x-5)}{(x+2)(x-4)}\ge 0$ $\qquad (d)\ \dfrac{2}{x-1}>\dfrac{3}{x+2}$

Teaching Notes:

- A sign chart can also be helpful in solving these inequalities.
- Using test numbers can be difficult if the roots are close together. The students will not always calculate correctly.
- If you are using a graphing calculator in class, then this is a good section to use that technology in the presentation. Graphic representation gives the students a clearer idea. The Table feature is a great way to calculate test values.

Answers:

1. $(a)\ (4,\infty)$ $\qquad (b)\ (-\infty,-3]\cup[-1,5]$ $\qquad (c)\ (-\infty,-4)\cup(4,\infty)$ $\qquad (d)\ (-\infty,-4]\cup[0,2]$

2. $(a)\ (-\infty,-2)\cup(2,\infty)$ $\qquad (b)\ (-4,\infty)$

$(c)\ (-\infty,-3]\cup(-2,4)\cup[5,\infty)$ $\qquad (d)\ (-\infty,-2)\cup(1,7)$

Mini-Lecture 4.5
The Real Zeros of a Polynomial Function

Learning Objectives:

1. Use the Remainder and Factor Theorems
2. Use the Rational Zeros Theorem to list the potential rational zeros of a polynomial function
3. Find the real zeros of a polynomial function
4. Solve polynomial equations
5. Use the Theorem for Bounds on Zeros
6. Use the Intermediate Value Theorem

Examples:

1. Find the remainder if $f(x)=x^4-3x^3+2x-4$ is divided by $(a)\ x-5 \quad (b)\ x+4$
2. Use the Remainder Theorem to determine whether the function $f(x)=3x^4-6x^3-11x^2+4x+6$ has the factor $(a)\ (x-3) \quad (b)\ (x+2)$.
3. Discuss the real zeros of $f(x)=5x^5-3x^4+2x^3+x^2-2x-5$.
4. For the function $f(x)=2x^4+5x^3+x^2+10x-6$, (a) list the potential rational zeros, (b) find the rational zeros.
5. Solve the equation $x^4-2x^3-8x^2+10x+15=0$.
6. Find a bound to the zeros of $f(x)=x^5-4x^4+2x^3-5x+2$.
7. Show that $f(x)=x^4+x^3-9x^2-3x+18$ has a root between -2 and -1.

Teaching Notes:

- This is a very important section for any student that will be taking a calculus course. Make sure the students understand the Division Algorithm for Polynomials since this will form the basis for this section.
- Using a graphing calculator can speed up the process for the Rational Zeros Theorem.
- Emphasize the Intermediate Value Theorem, as this is very important in calculus.

Answers:

1. (a) 256 (b) 436 2. (a) yes (b) no 3. Three or one positive real zeros. Two or none negative real zeros.

4. $(a)\ \frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$ $(b)\ \frac{1}{2}, -3$ 5. $x=3,-1,\pm\sqrt{5}$ 6. -6 and 6

7. $f(-1)=12>0,\ f(-2)=-4<0$

Mini-Lecture 5.6
Complex Zeros; Fundamental Theorem of Algebra

Learning Objectives:

1. Use the Conjugate Pairs Theorem
2. Find a polynomial function with specified zeros
3. Find the complex zeros of a polynomial function

Examples:

1. Find the remaining two zeros of a polynomial of degree 6 whose coefficients are real numbers and has the zeros $2, -3,\ 2i,$ and $1-2i$.

2. Find a polynomial of degree 5 whose coefficients are real that has the zeros $0, -2i,$ and $2+i$.

3. Find the complex zeros of the polynomial function $f(x) = 2x^4 - 5x^3 - x^2 - 5x - 3$

Teaching Notes:

- This section brings all of the theorems learned about zeros together.
- Help students see how the previous sections have pointed to this. Get them to see the "big picture", so to speak.
- A graphing calculator can really help them see the behavior of the polynomial.
- Show them the graph of 4$^{\text{th}}$ degree polynomial with only two real zeros, but point out that there are 4 roots. This is a way to make the introduction of the complex zeros easy for them to see. A simple example is $x^4 - 16$.

Answers:

1. $-2i, 1+2i$

2. $f(x) = a\left(x^5 - 2x^4 + 9x^3 - 8x^2 + 20x\right)$

3. $i, -i, 3, -\dfrac{1}{2}$

Mini-Lecture 5.1
Composite Functions

Learning Objectives:

1. Form a composite function
2. Find the domain of a composite function

Examples:

1. For $f(x) = 2x + 3$ and $g(x) = x^2 - 2x$, find
   $(a)\ (f \circ g)(2) \quad (b)\ (g \circ f)(2) \quad (c)\ (f \circ f)(-3) \quad (d)\ (g \circ g)(-1)$
2. Find the domain of $(f \circ g)(x)$ if $f(x) = \dfrac{2}{x-3}$ and $g(x) = \dfrac{1}{x+4}$.
3. Find the domain of $(f \circ g)(x)$ if $f(x) = \dfrac{1}{x+2}$ and $g(x) = \sqrt{x+1}$.
4. Show that $(f \circ g)(x) = (g \circ f)(x) = x$ for $(f)(x) = 3x - 5$ and $g(x) = \dfrac{x+5}{3}$.
5. Find functions *f* and *g* such that $(f \circ g)(x) = H(x)$ for $H(x) = \sqrt{x+5}$.

Teaching Notes:

- The concept of a composite function is not difficult for most students, but the algebraic manipulations that are necessary will be problematic to some.
- Students will struggle a little bit with the domain of a composite function if the functions are rational or radical functions. This will require some time and careful examples.
- Demonstrate that generally $(f \circ g)(x) \neq (g \circ f)(x)$, but that there are cases where $(f \circ g)(x) = (g \circ f)(x) = x$.. Tell them that when this is true there is a special relationship between the two functions that will be studied in future sections.
- If time permits, it would be good to go over the calculus application in Example 6, which is demonstrated in example 5 above. Emphasize the point that there is not a unique answer.

Answers:

1. $(a)\ 3 \quad (b)\ 35 \quad (c)\ -3 \quad (d)\ 3$
2. $\left\{x \mid x \neq -4, x \neq -\dfrac{11}{3}\right\}$
3. $\{x \mid x \geq -1\}$
4. $(f \circ g)(x) = (g \circ f)(x) = x.$
5. Answers can vary. One solution is $f(x) = \sqrt{x}$, $g(x) = x + 5$.

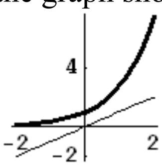
Mini-Lecture 5.2
One-to-One Functions; Inverse Functions

Learning Objectives:

1. Determine whether a function is one-to-one
2. Determine the inverse of a function defined by a map or a set of ordered pairs
3. Obtain the graph of the inverse function from the graph of the function
4. Find the inverse of a function defined by an equation.

Examples:

1. Determine whether the function $\{(2,-1),(3,2),(1,-1),(5,1)\}$ is one-to-one.
2. Find the inverse of the one-to-one function $\{(2,-1),(3,2),(1,3),(5,0)\}$.
3. For the graph shown, draw the graph of the inverse. The graph *y*=*x* is included to help.



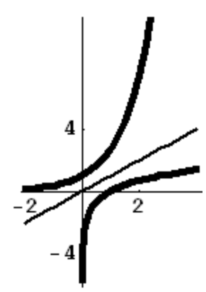
4. For the function $f(x)=x^2-2,\ x\geq 0$, find f^{-1}, state the domain and range of both functions, and graph the functions as well as the line *y*=*x*.
5. Find the inverse of the function $f(x)=\dfrac{x}{2x+1}$.

Teaching Notes:

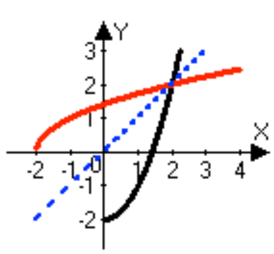
- One of the most common mistakes made is that students think $f^{-1}(x)=\dfrac{1}{f(x)}$. The fact that the -1 is a notation and not an exponent must be made clear.
- It is important that students understand $D_f=R_{f^{-1}}$ and $D_{f^{-1}}=R_f$. This can be demonstrated graphically for emphasis.

Answers:

1. No 2. $\{(-1,2),(2,3),(3,1),(0,5)\}$

3. 

4. $f^{-1}(x)=\sqrt{x+2}$; $D_f=R_{f^{-1}}=[0,\infty)$, $D_{f^{-1}}=R_f=[-2,\infty)$



5. $f^{-1}(x)=\dfrac{x}{1-2x}$