

3. Let  $S \subseteq \{0,1\}^k$   $|S| = n$ .

Prove that the following are equivalent

1. Let  $A$  be the  $n$ -by- $k$  matrix whose rows are the elements of  $S$ .  
Let  $C$  be the subspace generated by the columns of  $A$ .

$$\forall x, y \in C \quad x \neq y, \quad \frac{1-\epsilon}{2} \leq \frac{1}{n} \text{dist}(x, y) \leq \frac{1+\epsilon}{2}$$

2. For every  $\alpha \in \{0,1\}^k \setminus \{0\}$   
$$\left| \mathbb{E}_{S \in S} \left[ (-1)^{\sum_{i=1}^k \alpha_i S_i} \right] \right| \leq \epsilon$$

(such an  $S$  is called an  $\epsilon$ -biased set)

3. The Cayley graph  $G = \text{Cay}(\{0,1\}^k, S)$  has  $\lambda(G) \leq \epsilon$   
where  $\lambda = \max(|\lambda_2|, |\lambda_n|)$  and  $\lambda_i$  are the eigenvalues of  $M_G$ .